# *Corrections for Cluster-plot Slop*

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# *Outline*

- Sampling with cluster plots
- The slopover problem
- Correction methods:
  - Walkthrough
  - Walkabout
  - Vectorwalk
  - Reflection
- Discussion and questions

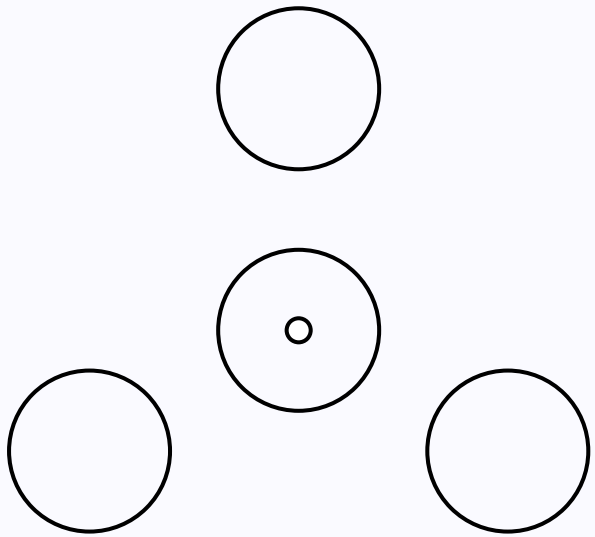
## Population, objects, and attributes

- Contiguous or non-contiguous tract of land, $\mathcal{A}$, with horizontal area $A$.
- Population comprises the infinitely many location points in $\mathcal{A}$.
- $\mathcal{A}$ contains $N \geqslant 0$ trees; $N$ is unknown.
- Tree $k$ has an attribute $y_k$.
- Total amount of attribute in $\mathcal{A}$ is:

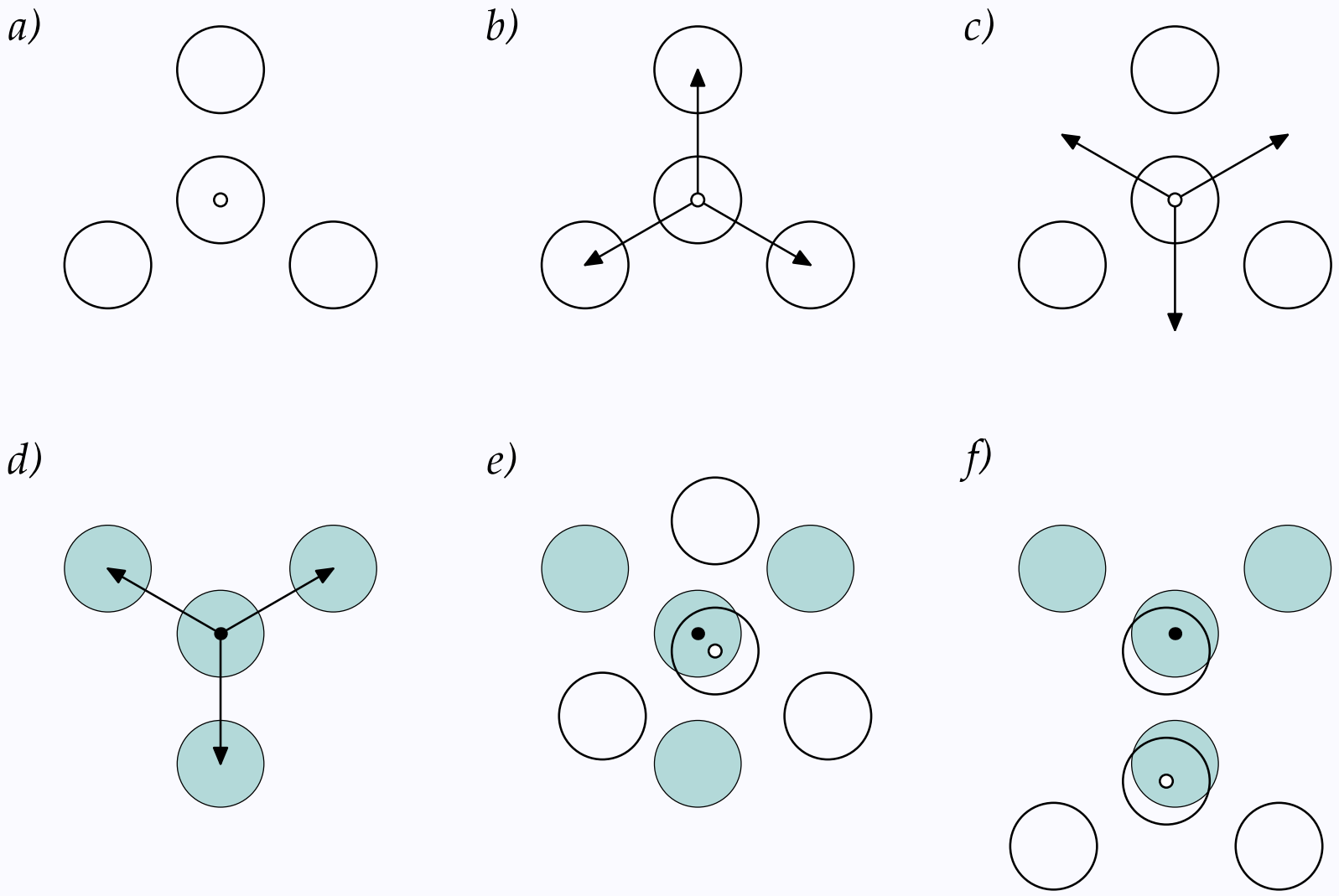
$$\tau = \sum_{k}^{N} y_k$$

Sampling:

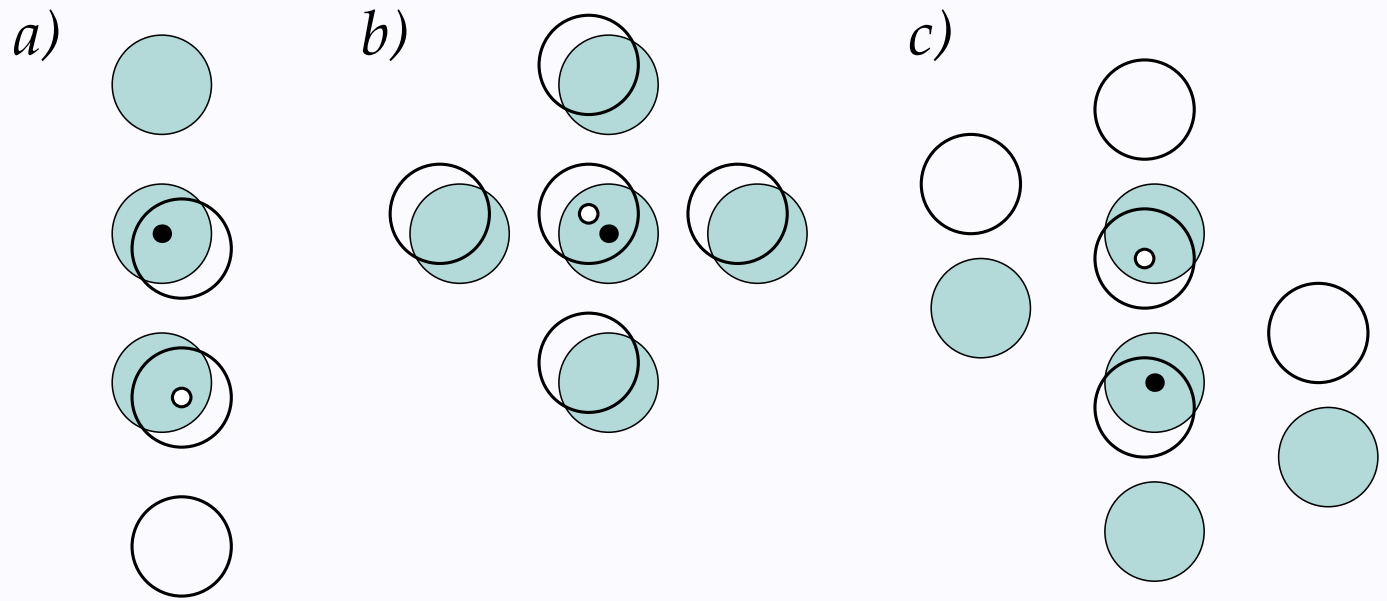
- Select $j$th sample point at $(x_j, z_j)$ uniformly at random (i.e., with probability density $f(x_j, z_j) = 1/A$).
- Anchor a cluster plot ($n$ subplots, each with area $a$) at the sample point $(\circ)$



Inclusion zone of $k$th tree ($I_k$):



# Radial symmetry



Attribute density ($\rho$):

- Attribute density in $I_k$:

$$\rho_k = \frac{\text{amount of attribute}}{\text{area of inclusion zone}} = \frac{y_k}{na}$$

- Attribute density at sample point $(x_j, z_j)$:

$$\rho(x_j, z_j) = \sum_{(x_j, z_j) \in I_k} \rho_k$$

Total attribute in $\mathcal{A}$ (sans slop):

$$\tau = \iint_{\mathcal{A}} \rho(x, z)\, \mathrm{d}x\, \mathrm{d}z = \sum_{k}^{N} y_k,$$

so we can estimate $\tau$ by Monte Carlo integration.

Monte Carlo estimator of $\tau$ (unbiased sans slop):

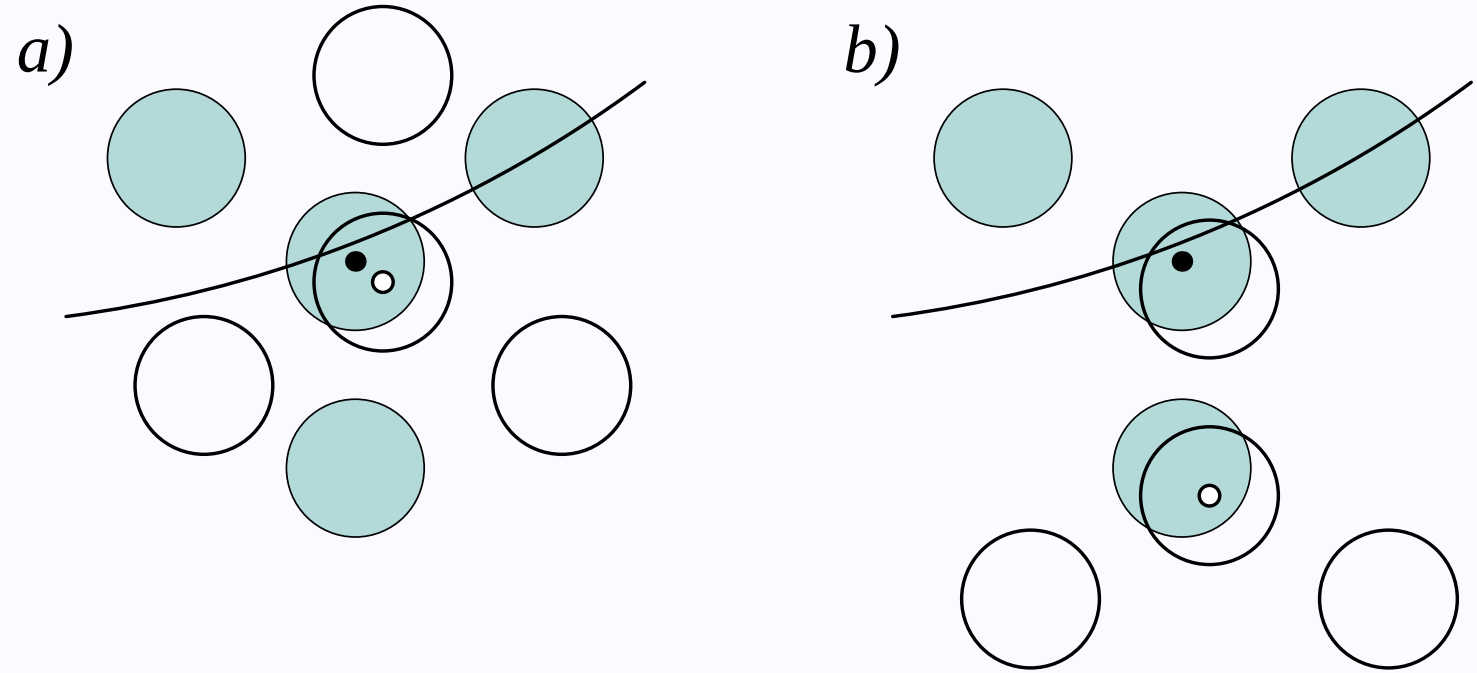
- $j$th sample point:

$$\hat{\tau}_j = \frac{\rho(x_j, z_j)}{f(x_j, z_j)} = A \sum_{(x_j, z_j) \in I_k} \rho_k$$

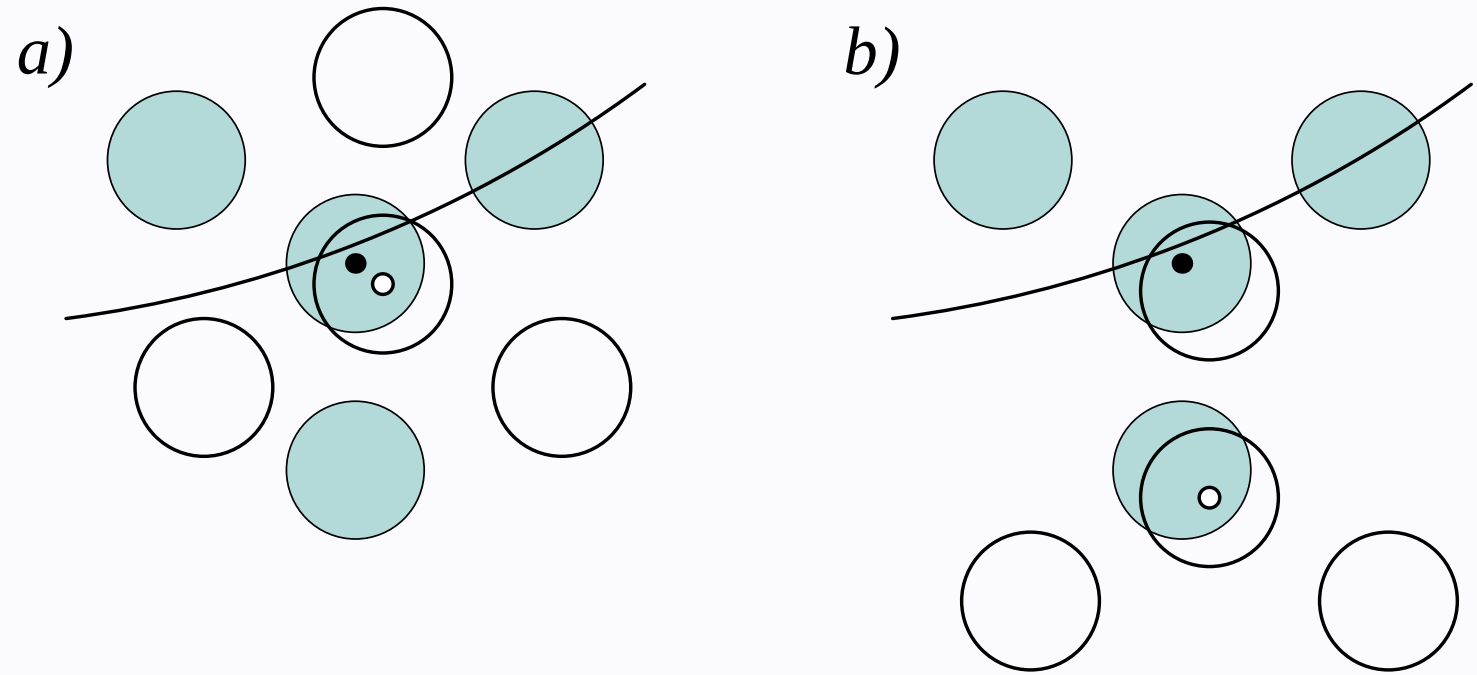
- $m$ sample points:

$$\hat{\tau} = \frac{1}{m} \sum_{j}^{m} \hat{\tau}_j$$

Problem: slop happens!

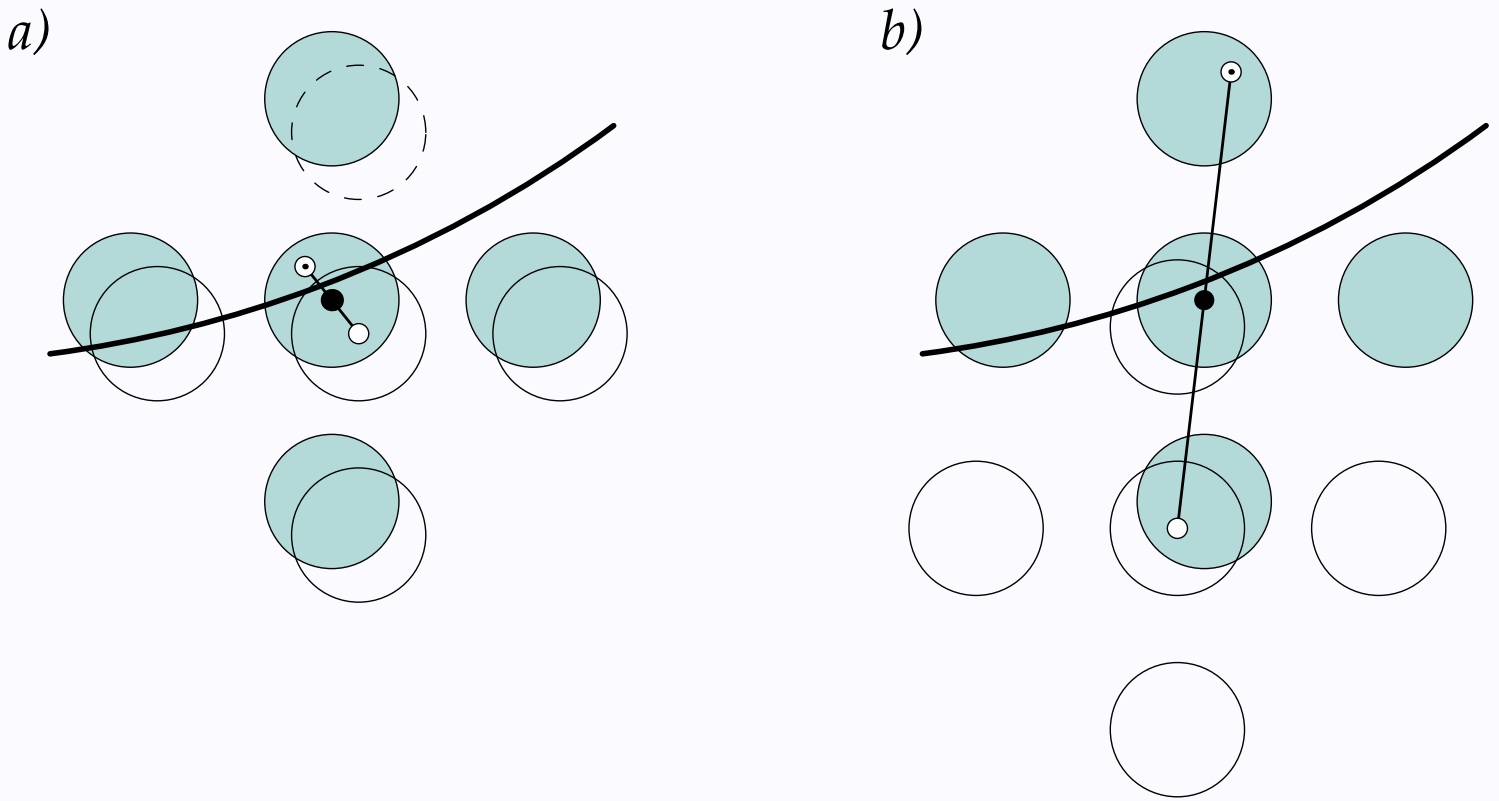


Problem: slop happens!

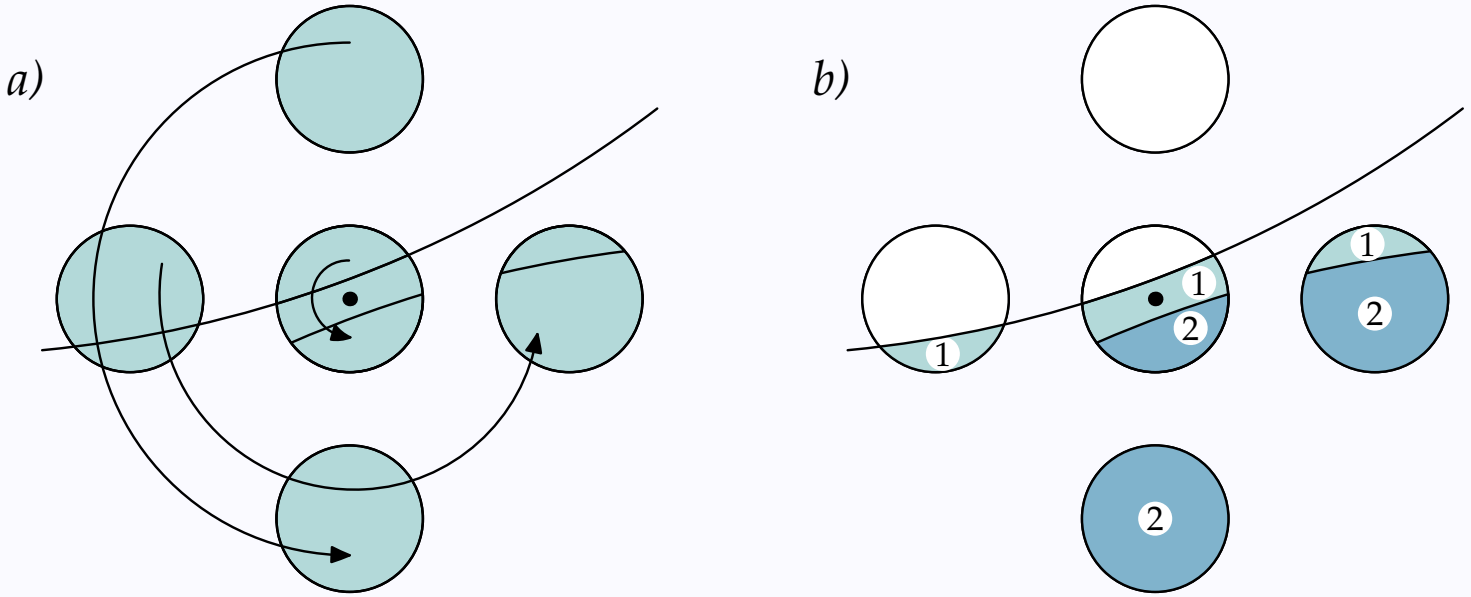



Solution: re-map attribute densities in $I_k$

# Walkthrough method



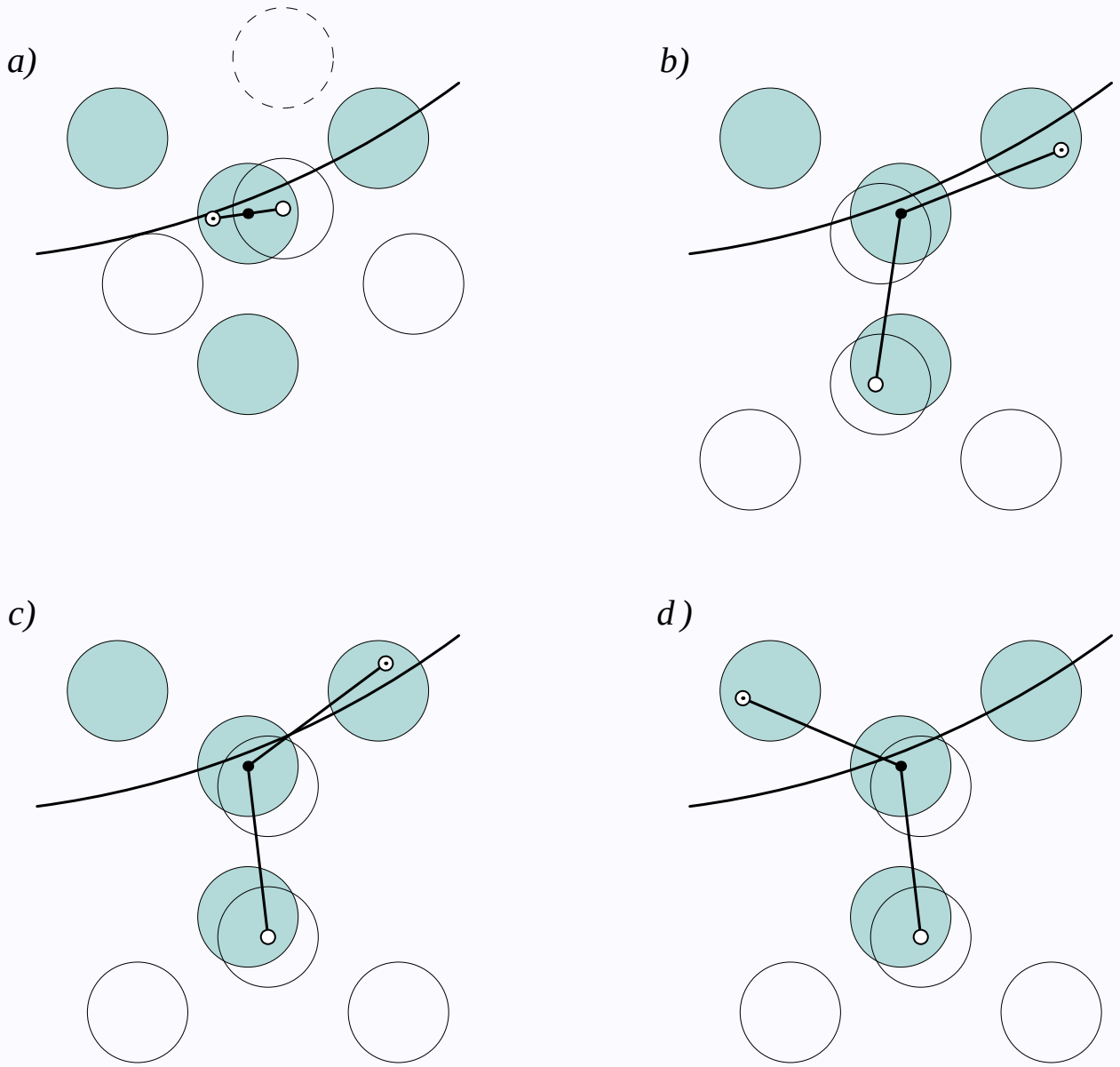
Walkthrough re-mapped attribute densities in $I_k$



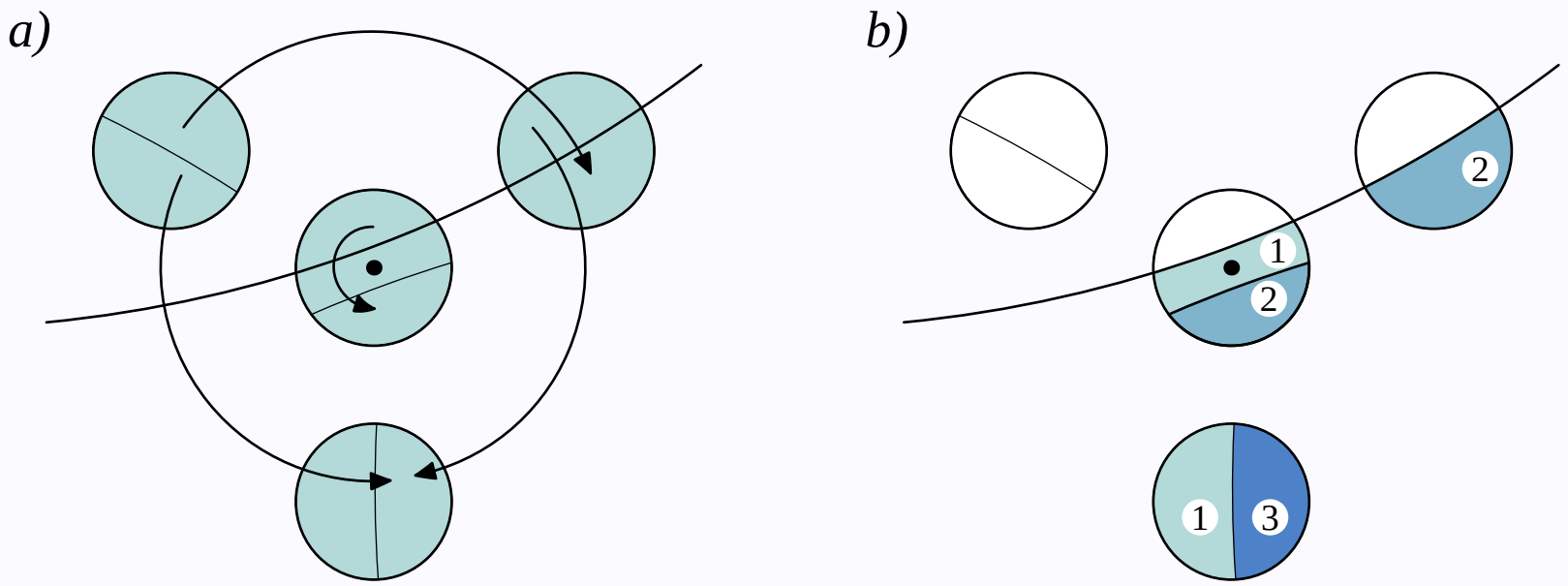
Walkthrough method – field instructions:

- Install satellite subplot if any part of it is in $\mathcal{A}$.
- Walk from $\circ \rightarrow \bullet \rightarrow \odot$.
- Double tally the tree if $\odot \notin \mathcal{A}$.
- Do for all trees between $\circ$ and boundary.

# Walkabout method



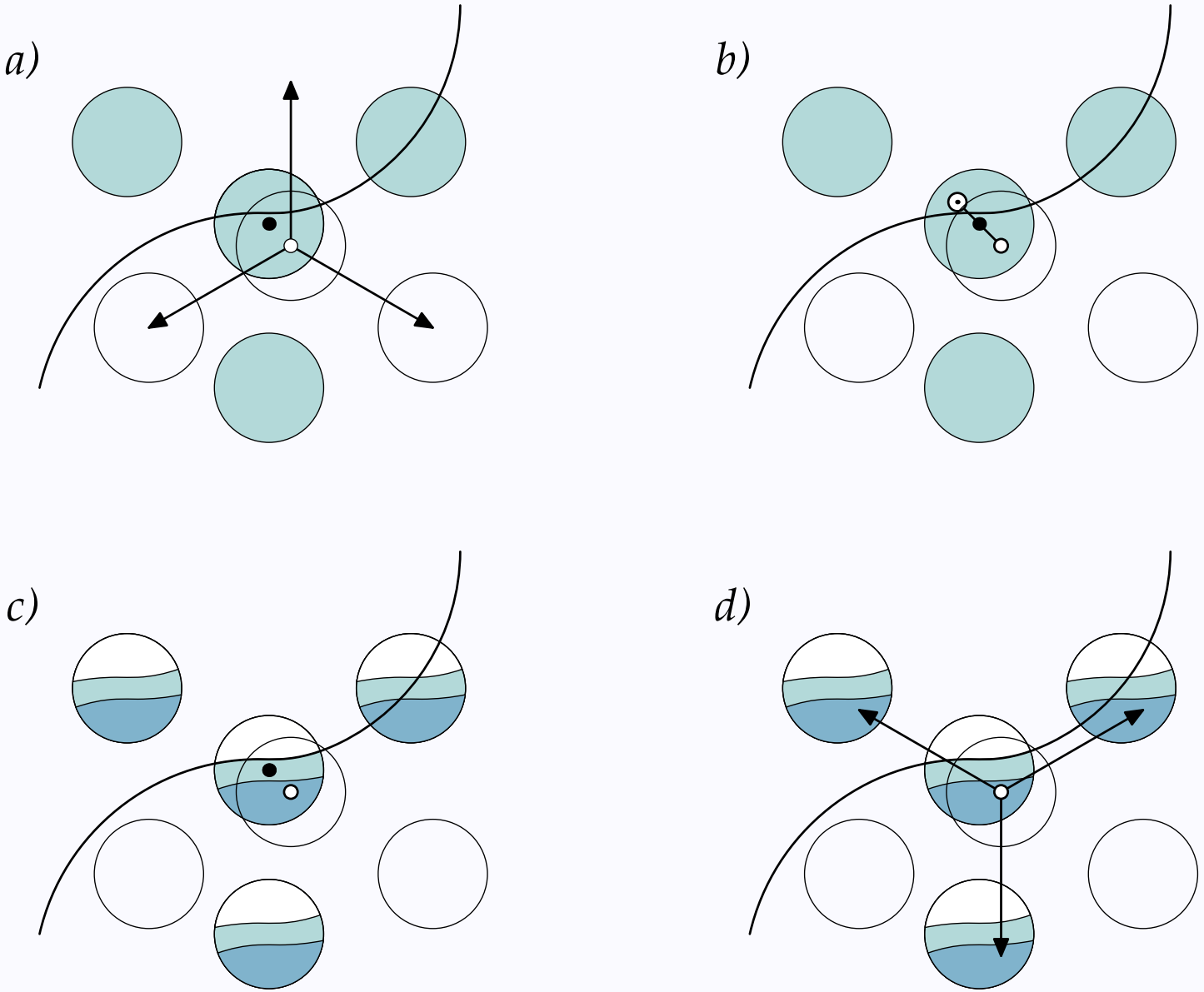
Walkabout re-mapped attribute densities in $I_k$



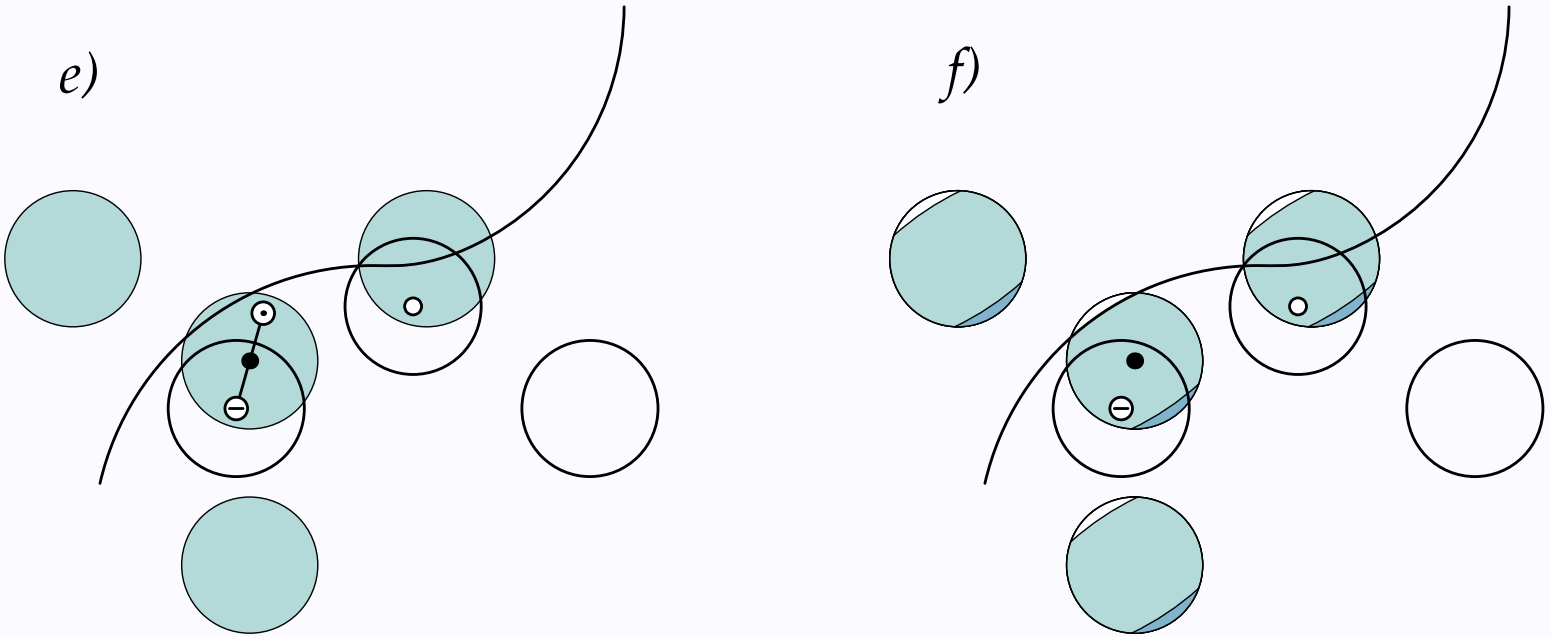
Walkabout method – field instructions:

- Install satellite subplot if any part of it is in $\mathcal{A}$.
- Map tree locations in each subplot.
- Map enough boundary points to spline a boundary line.
- Perform walkabout graphically or mathematically in the office.

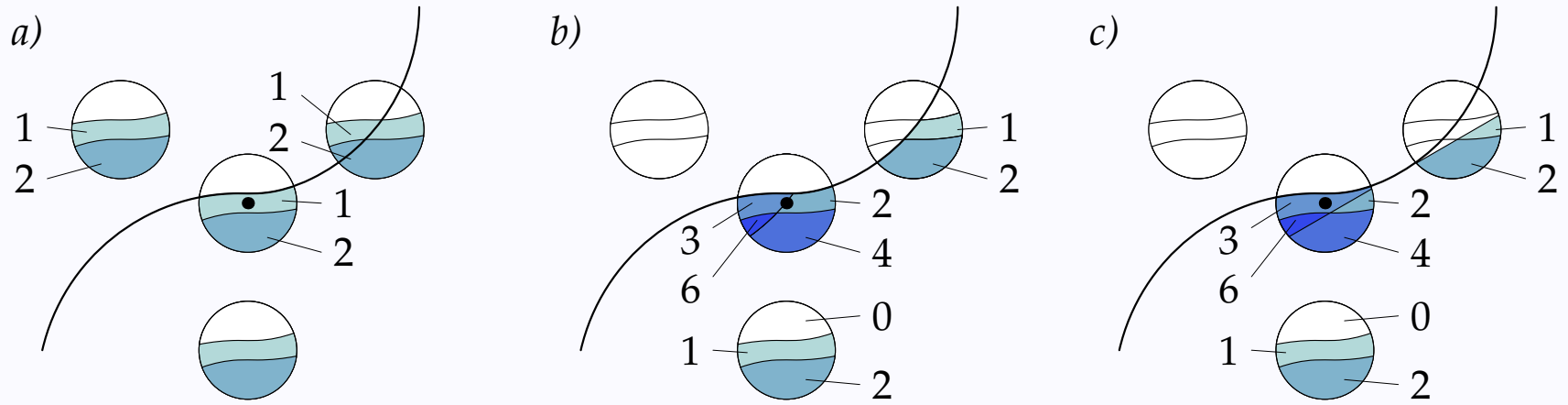
# Vectorwalk method – central subplot



# Vectorwalk method – satellite subplot



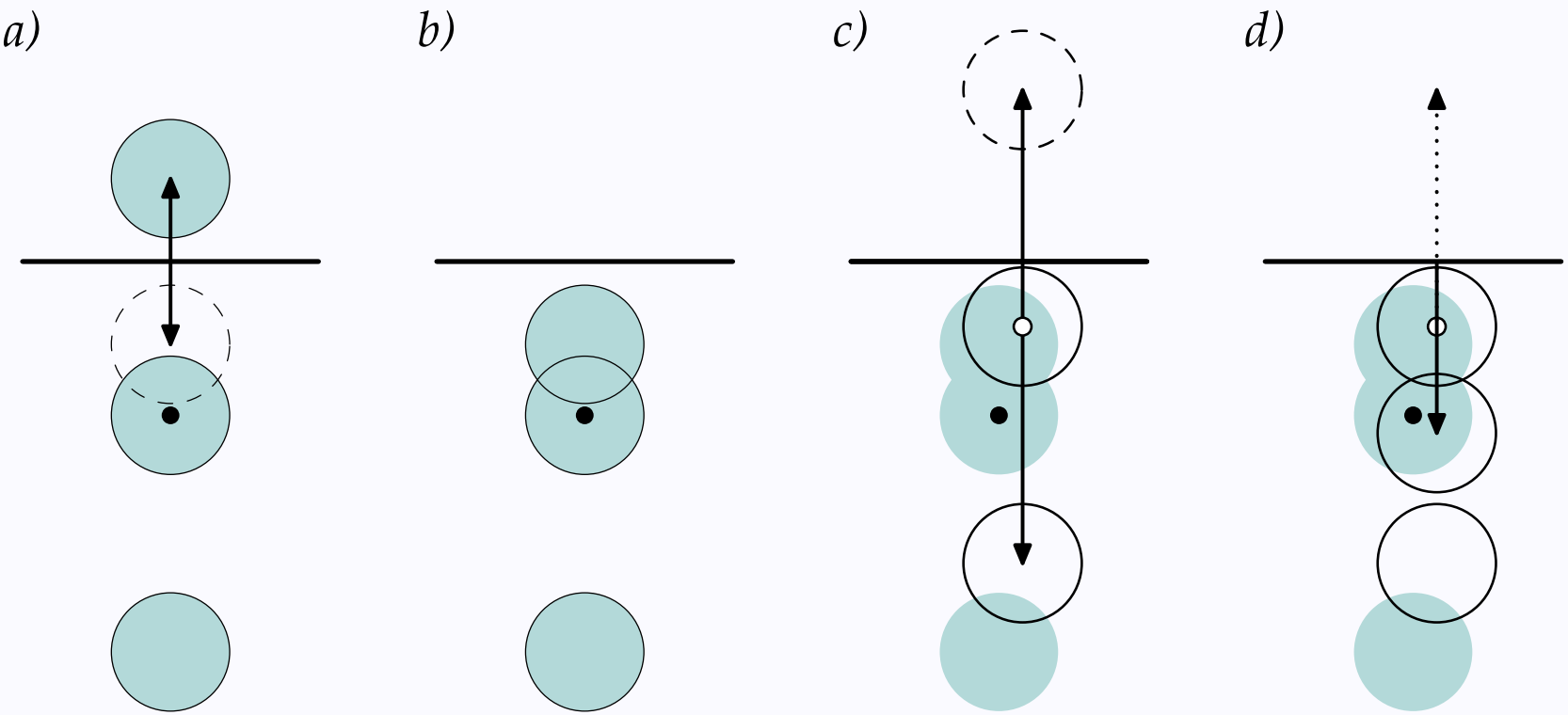
# Vectorwalk re-mapped attribute densities in $I_k$



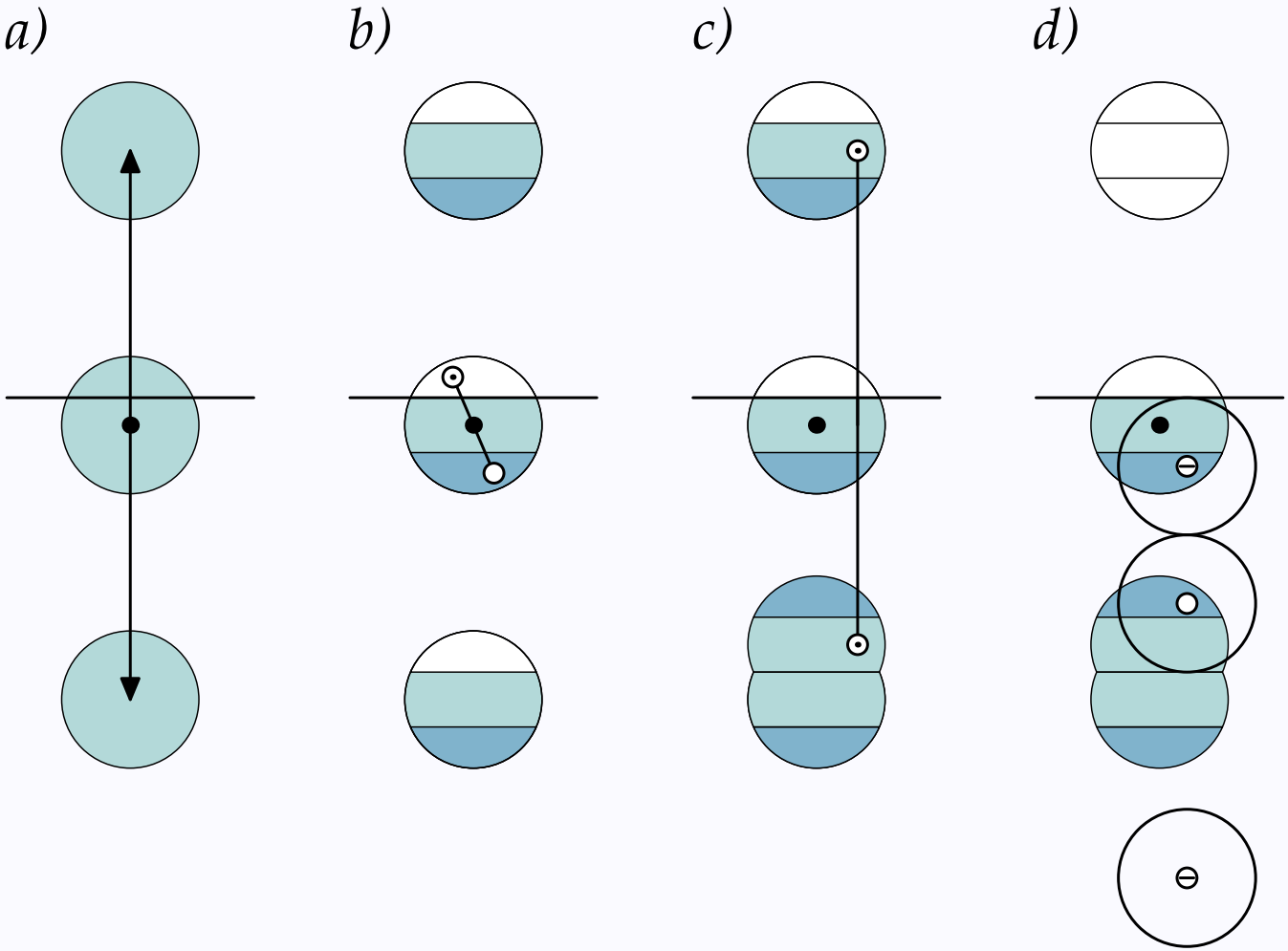
Vectorwalk method – field instructions:

- Install satellite subplot only if $\ominus \in \mathcal{A}$.
- Perform slop correction independently in each subplot (e.g., with the walkthrough method).
- The resultant single or double tallies are final for trees in satellite subplots, but preliminary for trees on the central subplot.
- Count the number $(c)$ of inverse vectors that terminate outside of $\mathcal{A}$. Multiply the preliminary tally of each tree on the central subplot by $(1 + c)$ to get the final tally.

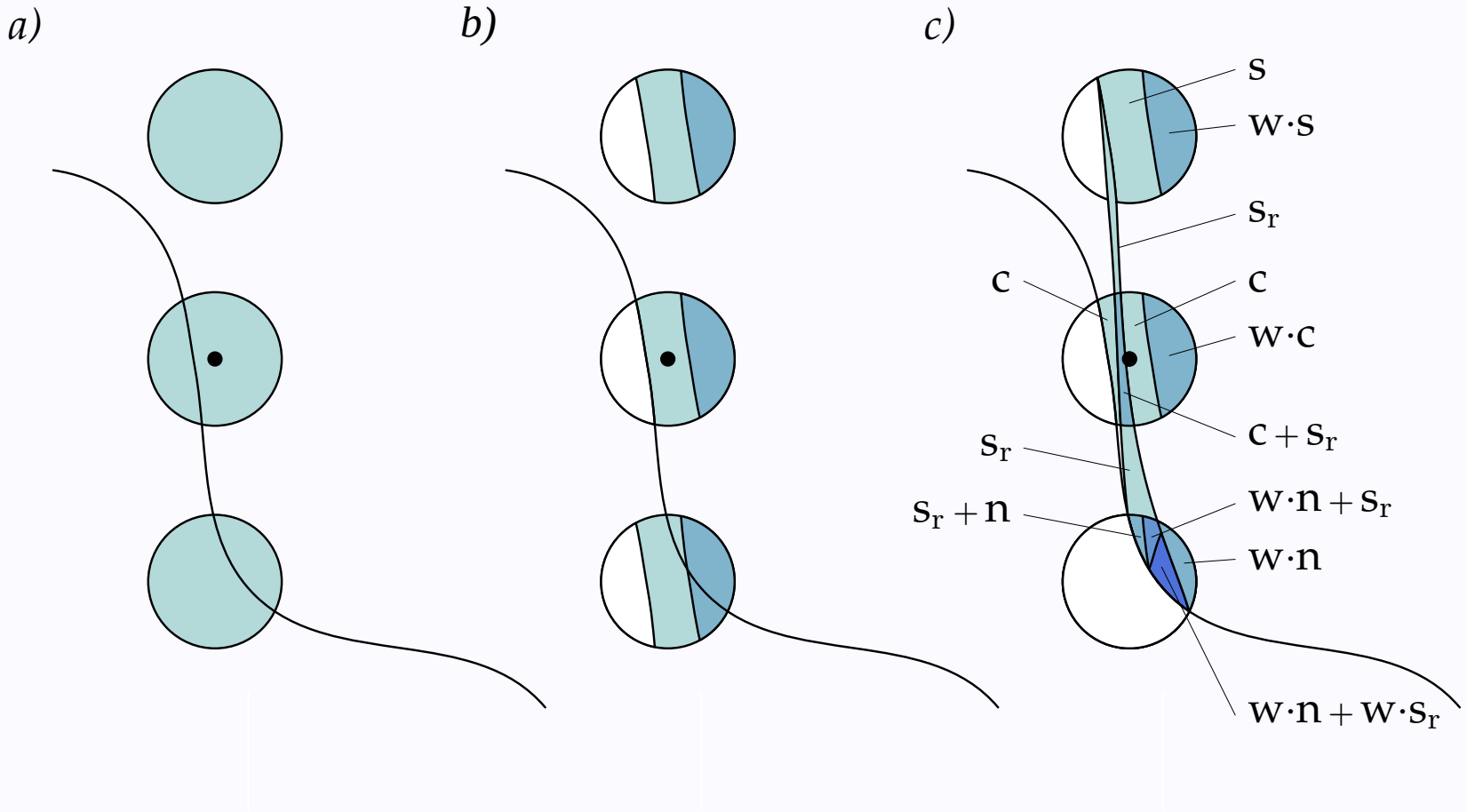
# Reflection method (for radial symmetry)



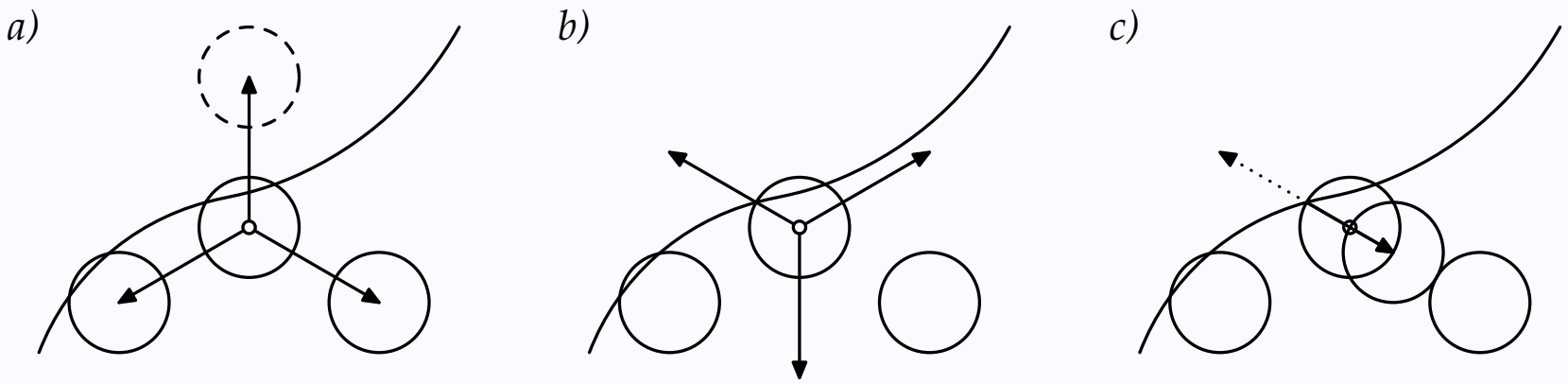
# Attribute densities re-mapped by reflection



# Reflection works with oblique and/or curved boundaries, too!



Reflection for radial asymmetry (e.g., the FIA design)



Reflection method – field instructions:

- **Radial symmetry**: fold back any direction vector at the boundary and install the satellite subplot at the terminal point in $\mathcal{A}$.
- **Radial asymmetry**: *do not* install a satellite subplot if its direction vector terminates outside of $\mathcal{A}$.

  Instead, fold back any inverse vector at the boundary and install a subplot where it terminates in $\mathcal{A}$.
- Perform slop correction independently in each subplot, e.g., with the walkthrough method.
- Trees may occur in more than one subplot and have different tallies in each.

## Corrections for cluster-plot slop

- Corrected attribute density of $k$th tree at $(x_j, z_j) \in I_k$:

$$\rho_k = \frac{y_k t_k(x_j, z_j)}{na},$$

  where $t_k = 1, 2, 3, \ldots$ is the correction factor or 'tally.'

- Estimator of $\tau$:

$$\hat{\tau}_j = A \sum_{(x_j, z_j) \in I_k} \rho_k$$

## Summary

- Walkthrough – radial symmetric cluster designs.
- Walkabout – FIA design and some other designs.
- Vectorwalk – Any design with a central subplot.
- Reflection (of direction vectors) – radial symmetric designs, asymmetric designs with random orientation.
- Reflection (of inverse vectors) – asymmetric designs with fixed orientation (e.g., FIA design).

*Any questions?*