Boundary Overlap Corrections for Segmented Line Transects

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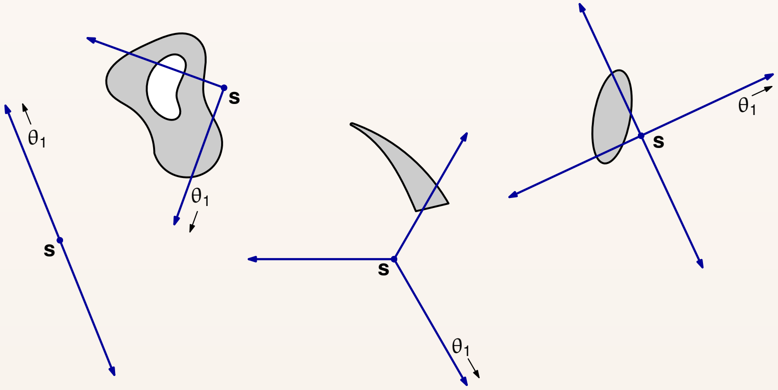
Outline

1. Line intersect sampling & segmented transects
2. Boundary overlap bias
3. Bias corrections
   - Symmetric radial transects
   - Asymmetric radial transects
   - Polygonal transects
4. Discussion

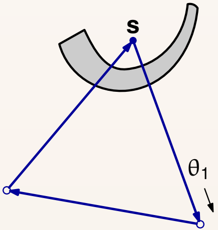
Segmented Transects
Radial

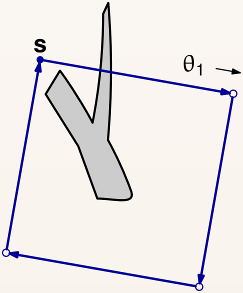
- $K \geqslant 1$ segments emanating from the sample point (**s**)



Segmented Transects
Polygonal

- $K > 2$ segments forming a closed figure at **s**





Notation

Population:

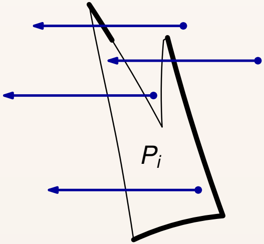
- N particles $\boldsymbol{P} = \{P_1, P_2, \ldots, P_N\}$ on T
- Target parameter: $\tau = \sum_{i=1}^{N} y_i$

Sample Design:

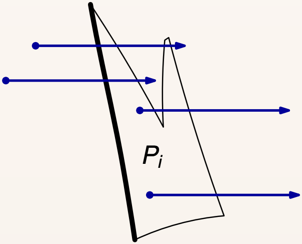
- $\mathbf{s}$ randomly located on T
- $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_K]$ with θ_1 selected:
  1. Arbitrarily, in advance of sampling; or,
  2. Uniformly at random from $[0, 360°]$

Selection Protocol

- t_{ik} is the number of times P_i is selected by segment k
- $t_{ik} = 1$ if segment k crosses the "leading edge":



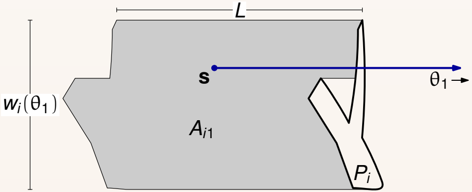
(a) $\theta_1 = 270^\circ$



(b) $\theta_1 = 90^\circ$

Inclusion Regions

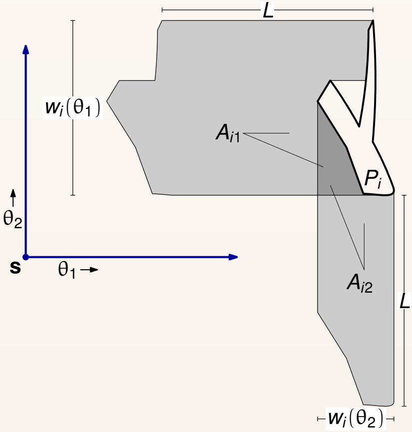
- Given θ_k, A_{ik} is inclusion region of P_i corresponding to segment k:



- $\Pr(t_{i1} = 1 | \theta_1) = \Pr(\mathbf{s} \in A_{i1} | \theta_1) = \dfrac{w_i(\theta_1) L}{T}$

Inclusion Regions
Radial Transects

- $t_{i\bullet} = \sum_{k=1}^{K} t_{ik}$
- $E(t_{i\bullet}|\boldsymbol{\theta}) = \sum_{k=1}^{K} \Pr(t_{ik}=1|\theta_k)$
$$= K\,\frac{\overline{w}_i(\boldsymbol{\theta})L}{T}$$



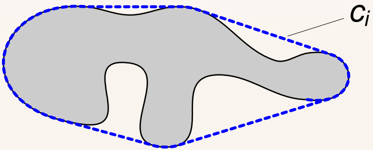
Estimation

Conditional on θ_1

- $\Pr(t_{ik} = 1 | \theta_k) = \dfrac{w_i(\theta_k)\, L}{T}$
- $E(t_{i\bullet} | \boldsymbol{\theta}) = K \dfrac{\overline{w}_i(\boldsymbol{\theta})\, L}{T}$

Unconditional

- $\Pr(t_{ik} = 1) = \dfrac{c_i\, L}{\pi T}$
- $E(t_{i\bullet}) = K \dfrac{c_i\, L}{\pi T}$



$$E[w_i(\theta_1)] = \frac{c_i}{\pi}$$

Estimation

Conditional on θ_1

- $\Pr(t_{ik} = 1 | \theta_k) = \frac{w_i(\theta_k)\, L}{T}$
- $E(t_{i\bullet} | \boldsymbol{\theta}) = K \frac{\overline{w}_i(\boldsymbol{\theta})\, L}{T}$

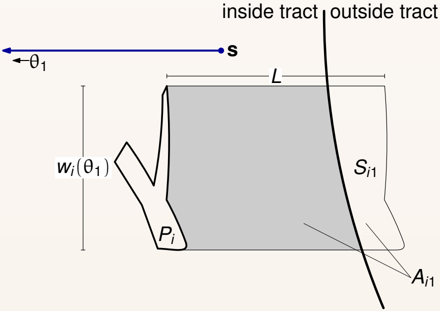
- $\widehat{\tau}_c = \sum_{i=1}^{N} \frac{t_{i\bullet}\, y_i}{E(t_{i\bullet} | \boldsymbol{\theta})}$

Unconditional

- $\Pr(t_{ik} = 1) = \frac{c_i\, L}{\pi T}$
- $E(t_{i\bullet}) = K \frac{c_i\, L}{\pi T}$

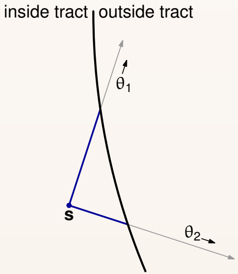
- $\widehat{\tau}_u = \sum_{i=1}^{N} \frac{t_{i\bullet}\, y_i}{E(t_{i\bullet})}$

- For P_i near the edge, and some θ_1:

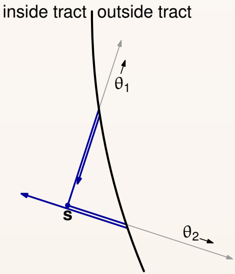


- $\Pr(t_{i1} = 1|\theta_1) < \dfrac{w_i(\theta_1)\,L}{T} \implies \begin{cases} E(\widehat{\tau}_c|\boldsymbol{\theta}) > \tau \\ E(\widehat{\tau}_u) > \tau \end{cases}$

Reflection Method
Radial Transects



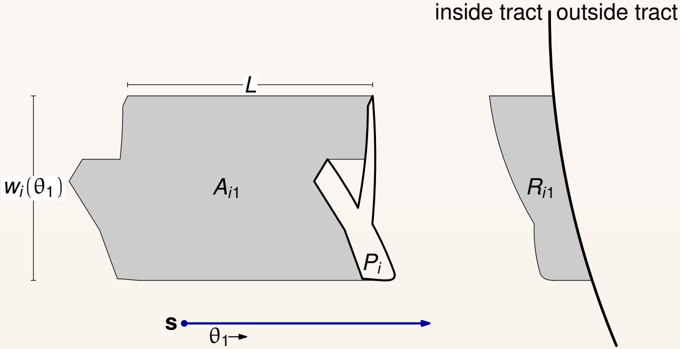
(a) Establish transect



(b) Reflect each segment
$t_{ik} = 0, 1, 2$

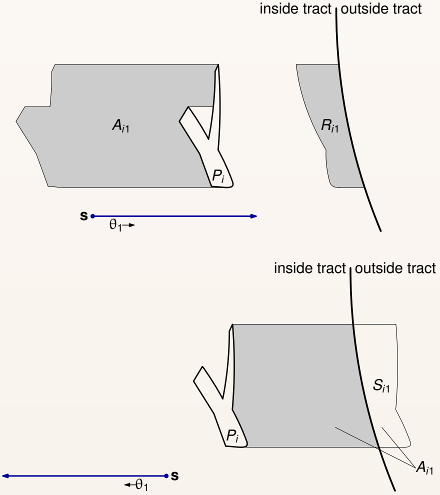
- cf. Gregoire & Monkevich (1994) *Environ & Ecol Stat* **1**: 219-226

Reflection Method
Radial Transects



Reflection Method

Radial Transects

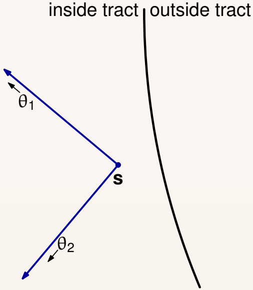


Reflection Method
Radial Transects

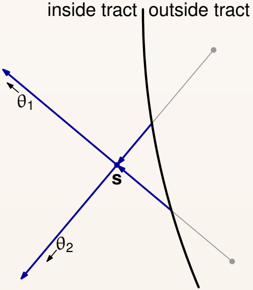
- $R_{i1} = S_{i2}$ if $\theta_2 = \theta_1 + 180^\circ$
- The reflection method is unbiased for:
  - Symmetric radial transects; *OR*
  - Randomly oriented radial transects
- The reflection method is *NOT* unbiased for:
  - asymmetric radial transects with fixed orientation

Walkback Method

Radial Transects



(a) Establish transect

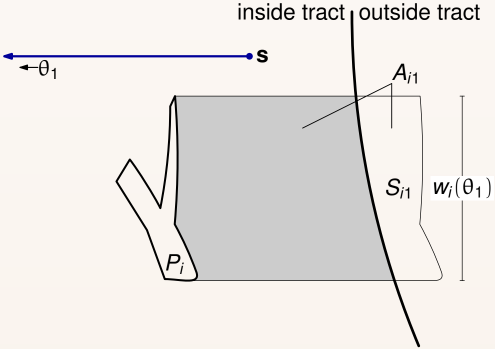


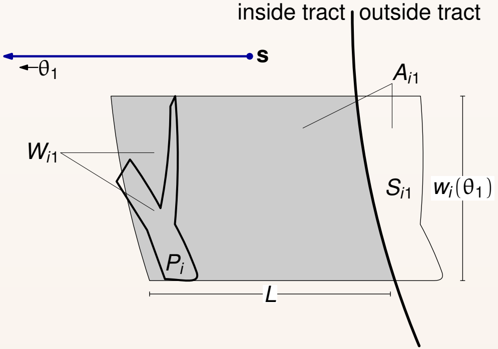
(b) Extend each segment

$t_{ik} = 0, 1$

Walkback Method

Radial Transects





- $W_{i1} = S_{i1}$
- Walkback method is unbiased for any radial transect

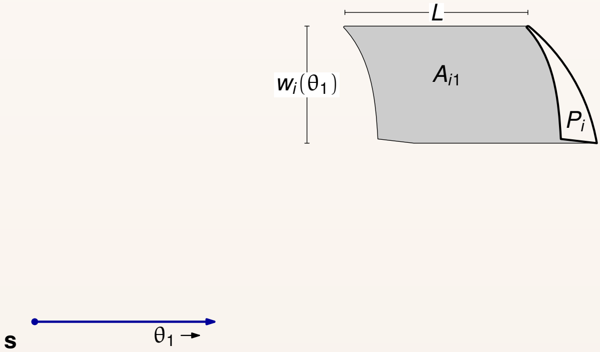
Summary
Radial Transects

To correct for boundary overlap bias:

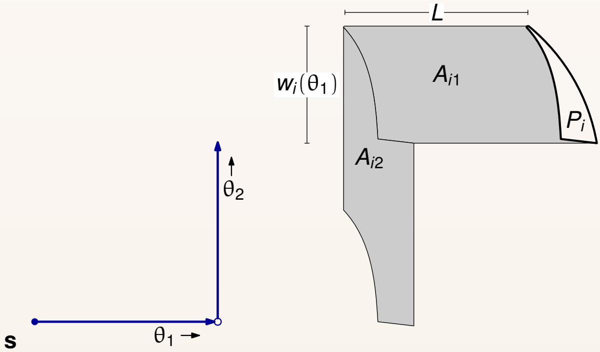
1. Symmetric *OR* randomly oriented radial transects
2. Asymmetric radial transects with fixed orientation

- Reflection method
- Walkback method

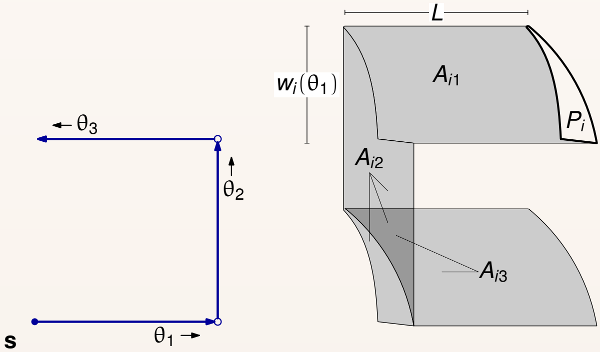
Inclusion Regions
Polygonal Transects



Inclusion Regions
Polygonal Transects

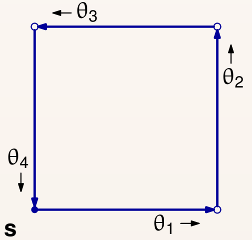


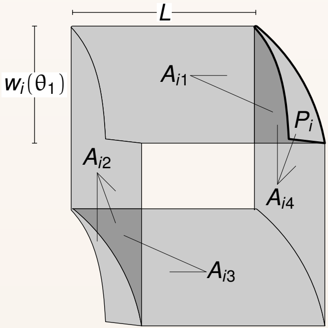
Inclusion Regions
Polygonal Transects



Inclusion Regions

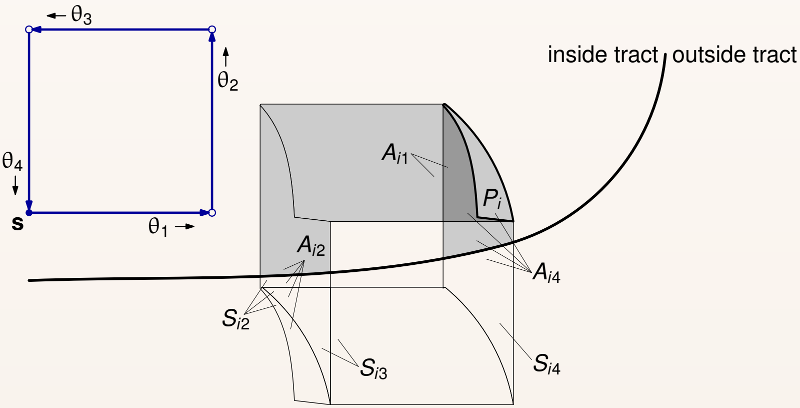
Polygonal Transects



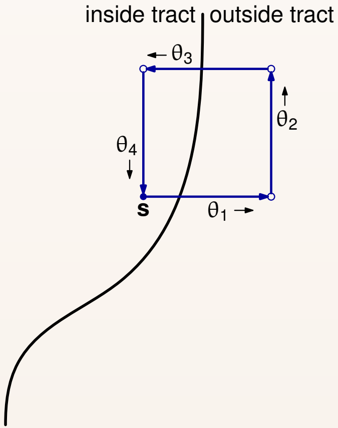


Boundary Overlap

Polygonal Transects

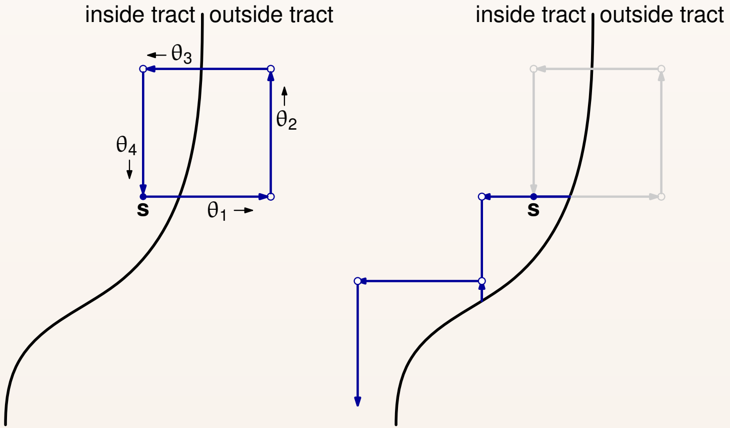


Reflection Method
Polygonal Transects



Reflection Method

Polygonal Transects



Summary

To correct for boundary overlap bias:

1. Symmetric *OR* randomly oriented radial transects
2. Asymmetric radial transects with fixed orientation
3. Polygonal transects with random orientation

- Reflection method
- Walkback method
- Reflection method

Or sample outside the tract.

Acknowledgments

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